Nonlinear homogenization (from 1983 to present days and beyond)

Martín I. Idiart
Facultad de Ingeniería, Universidad Nacional de La Plata
martin.idiart@ing.unlp.edu.ar

At sufficiently large loads, materials often exhibit nonlinear mechanical responses. Nonlinear homogenization theory aims at correlating the macroscopic response of microstructured materials with the nonlinear microscopic response of the constituents and their microstructural arrangement. This talk will present the historical development of a class of methodologies for addressing nonlinear homogenization problems by means of the concept of a linear-comparison medium. The presentation will be restricted to convex nonlinearities. The central idea of these methodologies is to exploit convex inequalities to identify `optimal' linearizations that allow the use of available linear bounds and estimates that account for microstructural morphology to generate corresponding non-linear bounds and estimates.

After introducing the variational characterization of the macroscopic or homogenized response in terms of an effective energy, the various methodologies to bound or estimate the effective energy are sequentially introduced and discussed. The sequence includes methodologies leading to secant, tangent, and generalized secant linearizations of the nonlinear responses. In passing, the exact expansion of the effective energy to second order in the heterogeneity contrast is presented. In all cases, sample results for two-phase composites exhibiting power-law responses are given. The presentation concludes with a summary on the state of the art of nonlinear homogenization methods and open problems.